Dr. Monteferrante

Dowling College
MTH 1022
Project II

by Georgi Todorov

I. Journal of meeting dates and times along with what transpired

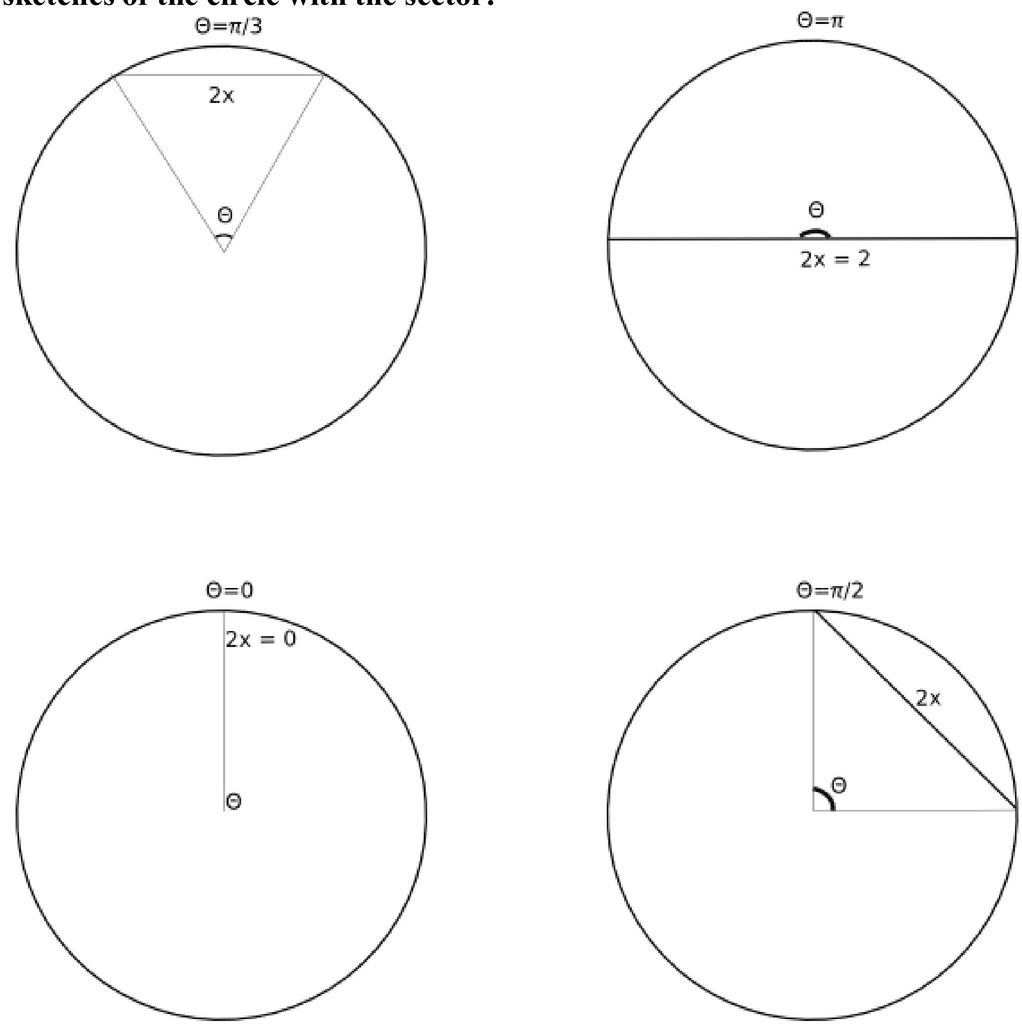
1. On 4th of April:
   Notes on how to solve the problem along with some basic sketches
2. On 7th of April:
   Found tools that plot graphs used later in the project. Plotted the needed graphs and did all math except the limit
3. On 11th of April:
   Solved the limit of the two functions. Compiled this document

II. Evaluation of the contributions, strengths and weaknesses of each member.

Took me too long to do the project!

III. Mathematical solution

1. 4 sketches of the circle with the sector:



2. x in each case:

a) when Θ = 0, x = 0

b) when Θ = π/3, x = $\frac{1}{2}$ because we have an equilateral triangle

c) when Θ = π/2, x = $\frac{\sqrt{2}}{2}$ Pythagorean theorem

d) when Θ = π, x = 1 (twice the radius)

3. A formula for S(x).

From $c^2 = a^2 + b^2 - 2abcos(\Theta)$ we have that $\Theta = \arccos(\frac{(c^2 - a^2 - b^2)}{(-2ab)})$

where c = 2x and a=b=r=1.

Finally we get $\Theta = \arccos(1-2x^2)$

The area of the total sector is :

$$S(x) = \frac{\Theta}{2} = \frac{(\arccos(1-2x^2))}{2}$$

4. A formula for T(x).

From $S(ABC) = \frac{1}{2} \times absin(\gamma)$ we can say that:

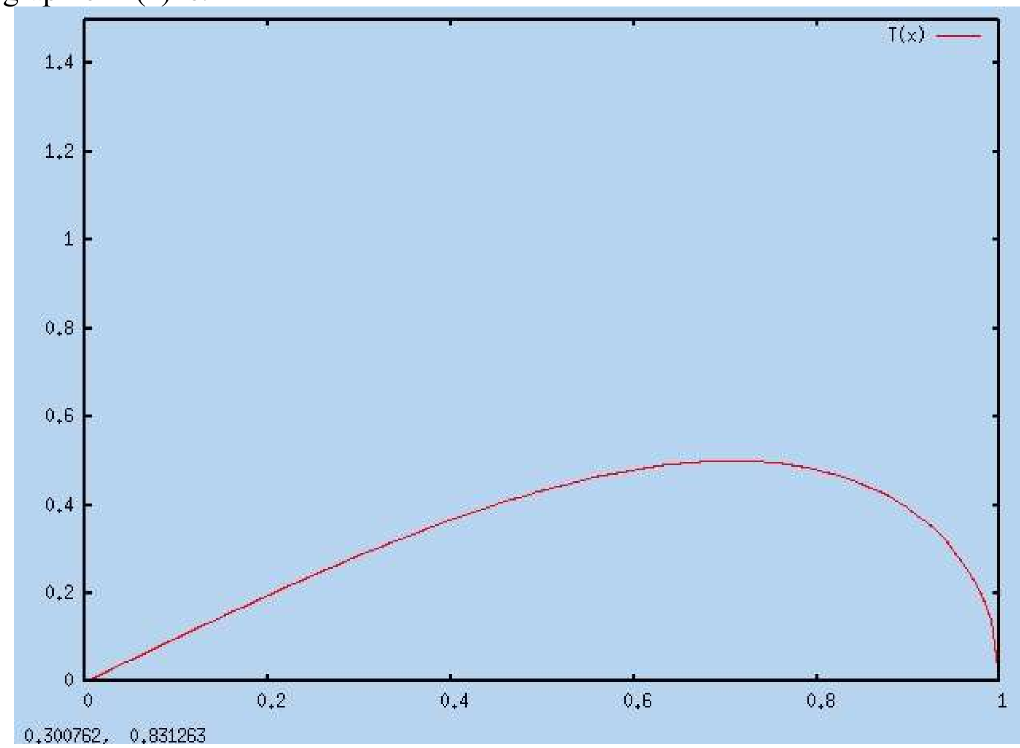
$$S(ABC) = T(x) = \frac{1}{2}\sin(\Theta) = \frac{1}{2}\sin(\arccos(1-2x^2))$$ but:

$$\sin(\arccos(x)) = \sqrt{1-\cos^2(\arccos(x))} = \sqrt{1-x^2}$$ so for T(x) we have that:

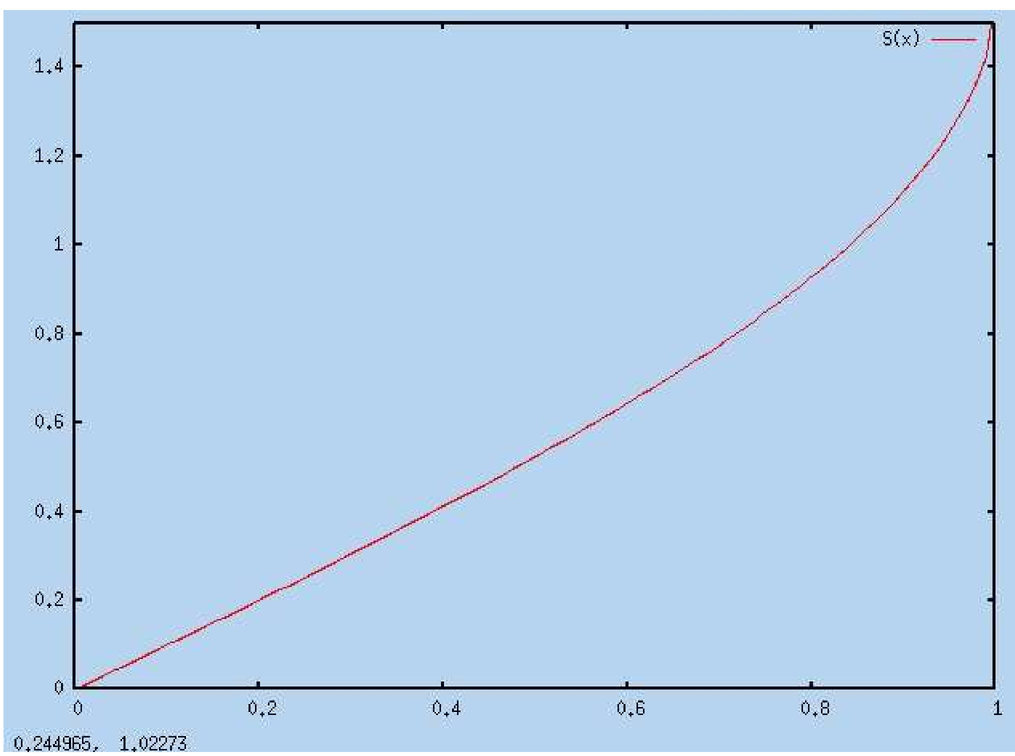
$$T(x) = \frac{1}{2}\sqrt{1-(1-2x^2)^2} = \frac{\sqrt{1-1+4x^2-4x^4}}{2} = \frac{\sqrt{(2x)^2(1-x^2)}}{2} = x \times \sqrt{1-x^2}$$

5. The 3 functions: S(x), T(x) and S(x)/T(x) for 0 < x < 1

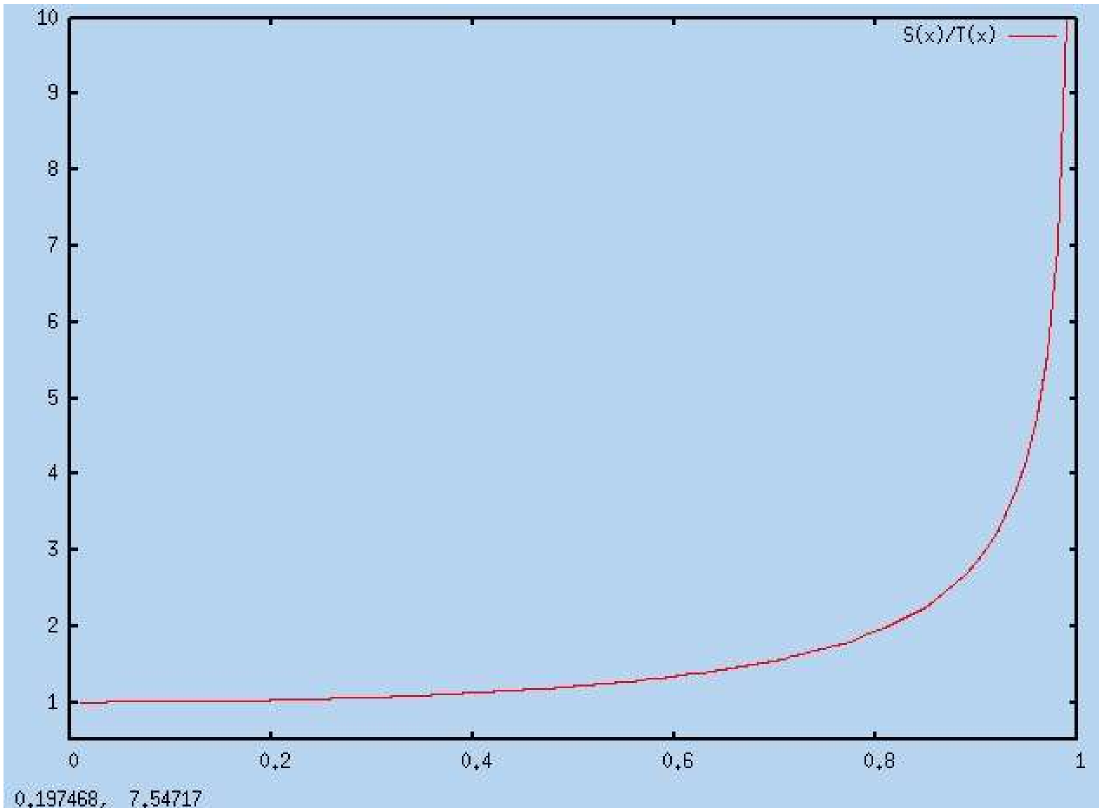
The graph of T(x) is:



The graph of S(x) is:



The graph of S(x)/T(x) is:



6. The Limit

$$\lim_{x\to 0}\frac{S(x)}{T(x)}=\lim_{x\to 0}\frac{\arccos(1-2x^2)}{2x\sqrt{1-x^2}}=\lim_{x\to 0}\frac{4x}{\sqrt{1-(1-2x^2)^2}}\div\left(2\sqrt{1-x^2}+\frac{2x^2}{\sqrt{1-x^2}}\right)$$

$$\lim_{x\to 0}\frac{4x}{\sqrt{1-(1-2x^2)^2}}\div\frac{2(1-x^2)+2x^2}{\sqrt{1-x^2}}=\lim_{x\to 0}\frac{4x}{\sqrt{1-(1-2x^2)^2}}\div\frac{2}{\sqrt{1-x^2}}$$

$$\lim_{x\to 0}2x\sqrt{\frac{1-x^2}{1-(1-2x^2)^2}}=\lim_{x\to 0}2x\sqrt{\frac{1-x^2}{1-1+4x^2-4x^4}}=\lim_{x\to 0}2x\sqrt{\frac{1}{4x^2}}$$

$$\lim_{x\to 0}\frac{2x}{2x}=1$$

7. Why the results of questions 5 and 6 make sense geometrically?

The Graph of S(x) over F(x) shows what the limit will be as x goes to 0. We can see how as X goes closer and closer to 1, the values of y go closer and closer to 1. In general graphs can be used to evaluate limits for every value of x. It is just necessary to trace the values along the line. For example using the same graph we can say that the same limit with x going to 1 will be infinity.

If we take a look at the graph of S(x) we will see that as x goes to 0 y goes to 0 too. The same happens with T(x). That means that the limit of the one over the other will be 1 (since x is never 0).